

ART. VI.—*The Technique of the Nanson Preferential Majority System of Election.*

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I.

In 1925 a joint conference of the University Council, the Standing Committee of Convocation, and the University Association was appointed to consider the question of methods of voting at University elections. The Conference, having decided at its first meeting in favour of a system requiring election by a majority, considered that, of the different majority methods available, that devised by Professor Nanson (*Trans. Roy. Soc. Vic.*, xix., p. 197, 1882) was the best, but that the labour involved in the counting, except when the number of candidates was small, might be excessive.

Mr. Picken (Council) in a memorandum drew the attention of the conference to an alternative method of tabulating the votes given by Mr. G. Hogben (*Trans. N.Z. Inst.*, xlvi., p. 304, 1913), and at the second meeting Mr. Le Couteur (Association) and Dr. Baldwin (Standing Committee) both expressed the opinion that the Nanson method with the Hogben tabulation could be carried out without undue labour, provided that the number of candidates was not very large. A sub-committee was appointed to conduct a test election, and the result showed that this opinion was justified.

At the third meeting it was suggested that, as a first step in an election with a large number of candidates, the number should be reduced on a count of first preferences by the rejection of those at the bottom of the list or the election of those at the top or by both methods, and Dr. Baldwin was asked to draw up a memorandum embodying these suggestions.

This was considered at the fourth meeting, when it was resolved to recommend that the Nanson method of voting, with a generalized form of the Hogben tabulation, should be adopted for the next election, and Mr. Phillips (Standing Committee) and Dr. Baldwin were appointed to draw up the necessary Statute. This draft statute, with some verbal alterations suggested by Sir Leo Cussen, was finally adopted by the Council and the Standing Committee.

In the course of these meetings and elsewhere, a technique was evolved, which was tested at the University elections last Decem-

ber. The experience gained there has shown that the method is a thoroughly practical one, and does not involve an undue amount of labour provided the number of candidates does not exceed 10.

II.

No critical examination of the different methods of voting is here attempted—that has already been fully done in Nanson's paper cited above. The present paper is a detailed description of a method of carrying out Nanson's system in its most general form, allowing the voter to indicate preferences for as few or as many candidates as he pleases, and to bracket two or more candidates if he so desires. It is thus an extension of Hogben's paper, where the tabulation is for a single member electorate only and incomplete papers and bracketing are not considered. Actually such papers are dealt with almost as easily as other papers, so that any expression of preference by a voter will be recorded and have due weight in the final result.

In one detail only is the Nanson system departed from. In that system all candidates who are not above the average are rejected *en bloc*; here, as in the Trinity College Dialectic Society's elections (Nanson, *loc. cit.*, p. 217), the lowest only is excluded. In the original system this short cut meant a considerable saving of time, in the present method the extra time involved in carrying out the more rigorous procedure is quite negligible—a matter of a very few minutes only.

III.

RULES.

(*Melbourne University Calendar*, 1926, Statute 34, Division 1, Sections 21 to 26.)

1. The Voter shall indicate the order of his preference by writing numbers on his ballot paper opposite the names of all or some or one of the candidates. A number opposite the name of a candidate shall indicate a preference for that candidate over all candidates opposite whose names a higher number or no number is written, and the same number or no number opposite the names of two or more candidates shall indicate that the voter considers these candidates of equal merit.
2. The number of preferences for each candidate over each other candidate shall be ascertained. In each case where on a voting paper no preference is expressed as between two candidates, half a preference is to be credited to each of the two candidates. Where no preference is expressed as between more than two candidates, the candidates so bracketed shall be dealt with two at a time. The number of preferences shall be arranged in tabular form in which one column (ver-

- tical) and one row (horizontal) are assigned to each candidate, the number of preferences (for instance) for candidate P over candidate Q being written down in column P, row Q.
3. The numbers in each column shall be summed. The column with the lowest sum and the corresponding row shall be excluded, and the remaining numbers in each column shall again be summed. The column with the lowest sum at this stage and the corresponding row shall be excluded, and this process of summing and exclusion shall be repeated until only two columns are left. Of the candidates to whom these columns refer that one who has the majority of preferences over the other shall be declared elected.
 4. If a further vacancy is to be filled, the column and row assigned to each elected candidate shall be excluded, and the process of election carried out in precisely the same manner as before.
 5. If at any stage two columns (for instance, those assigned to B and C) have the same sum, and there is no other column with a lower sum,¹ then the column C shall be excluded if B has a majority of the preferences as between B and C, but if B has exactly half of the preferences as between B and C, the Returning Officer shall decide which column is to be excluded. If, at any stage, three or more columns have the same sum, and there is no other column with a lower sum, the Returning Officer shall decide which column is to be excluded.
 6. At any stage of the scrutiny the Returning Officer may adopt any modification which is the mathematical equivalent of the portion of the process for which it is substituted.

IV.

These rules may be used as they stand, but the labour of tabulation may be considerably shortened by a suitable arrangement of the work and by variations of the procedure which, however, can be shown by strict mathematical reasoning to lead to the same result, and are therefore allowable under Rule 6. Details of the procedure are given in the following instructions.

INSTRUCTIONS.

(The process described in a sentence or paragraph following an * is a mathematical equivalent of one laid down in the preceding rules.)

1.—An alteration has been made here, to provide more explicitly for the case where the equality occurs when all columns but those with equal sums have been excluded.

1. Sort the voting papers according to first preferences, and count the number of first preferences allotted to each candidate.

*For each paper where any number, p , of candidates are placed equal with a preference ranking as first, $1/p$ is to be credited to each of the candidates so placed.

*If the number of first preferences received by any candidate exceeds half the number of voting papers, that candidate is placed first at once, and is excluded from all further counts and tabulation.

2. If any candidate is elected under (1) redistribute the voting papers on which he is given first or equal first preference according to the preferences ranking as first preferences among the remaining candidates, and determine the total number of preferences ranking as first credited to each.

*For each paper where any number q of candidates are placed equal with a preference which ranks as first at this stage, $1/q$ is to be credited to each of the candidates so placed.

*If after the redistribution the number of preferences ranking as first preferences received by any candidate exceeds half the number of voting papers, that candidate is placed second, and is excluded from all further counts and tabulation. This process may be continued as far as possible.

3. (*For occasional use only.*) Sort the voting papers according to last preferences (candidates against whose names no number is written rank last), and count the number of last preferences allotted to each candidate.

*For each paper where p candidates are placed equal last, $1/p$ is to be debited against each of the candidates so placed.

*If the number of last preferences received by any candidate exceeds half the number of voting papers, that candidate is placed last, and is excluded from all further counts and tabulation.

4. To the candidates who remain after the completion of the above processes allot rows and columns² according to the count of first preferences, starting with the highest in the *top left-hand* corner, referred to the recording scrutineer.
5. The scrutineers work in pairs. Any pair, A and B say, deals first with a group of papers on each of which the first preference has been given to the same candidate, S say, and to no other. A has the voting papers

2.—The size of the tabulation sheet should not exceed 24 inches by 20 inches. If each column is 2 inches wide, and each row $1\frac{1}{2}$ inches wide, 200 preferences can readily be recorded in each space, and so 400 voting papers can be tabulated on the sheet.

and a number of strips. He fastens a strip over the row assigned to S, *and B writes the number of voting papers in the group in each of the spaces of the column assigned to S, above the diagonal through the top left-hand corner. Taking the first voting paper, A now calls out the name of the candidate with the second preference, and places a strip³ on the corresponding row, *while B puts a stroke in each of the uncovered spaces above the diagonal in the corresponding column. The third, fourth, etc., preferences are dealt with in a similar way.

Where no preference is indicated as between two or more candidates, A calls out (in the order in which they appear in the tabulation) the names of such candidates. Taking the row of the first such candidate, B puts a dot in the columns corresponding to the second, third, etc., such candidates; then taking the row of the second such candidate, he puts a dot in the columns corresponding to the third, etc., such candidates; and so on. This having been done, strips are placed over the rows in turn, and strokes put in the uncovered spaces as before.

When all the rows have been covered, the strips are gathered by A, and a similar process is gone through with each of the voting papers in turn, until the group is finished. Another group is then dealt with in a similar manner, and so on until all the papers have been tabulated.

Where the remaining candidates are numbered on a voting paper in the order in which their names appear on the tabulation, it will be found that no further strokes will need to be put on the tabulation for that paper.

6. As the treatment of voting papers with bracketing and those that are incomplete is slightly different from that of other papers, it is well to have all such voting papers tabulated by a special pair of scrutineers. If A and B, dealing with papers on which S has first preference, transfer any voting papers to the special pair, B must be careful to see that these papers have not contributed to any of the entries in his tabulation. If the preferences on a voting paper have been partially recorded before the bracketing is noticed, it is inadvisable to transfer the voting paper to the special scrutineers.
7. At the close of the tabulation, the number of strokes in each space is counted. To this is added half the number of dots in the same space. The results from each

3.—This use of strips was suggested by Mr. Le Couteur. The strips should be of cardboard, stiff enough to be placed in position with one hand, somewhat narrower than the rows, and long enough to reach across the sheet.

pair of scrutineers are added together, and thus the total number of preferences of each candidate over each candidate before him on the tabulation is obtained. *These are written on a similar table, and the spaces below the diagonal are filled in by subtracting the number in the complementary space above the diagonal from the total number N of voting papers. This completes the tabulation.

Note.—If the counting be carried on at several centres, the third part of (1) and the whole of (2) and (3) are inapplicable. The order of columns and rows will probably be different for the different centres, but when the tabulations are all received at the head centre it will be a simple matter to combine them into a single tabulation for the whole electorate.

8. Inspect the table and ascertain—

- * (a) if in any column each number is greater than $N/2$. In such case the candidate to whom that column refers is placed first, and his row and column are covered with fastened strips.
- * (b) if now in any column each uncovered number is greater than $N/2$. In such case the candidate to whom that column refers is placed second, and his row and column are covered with fastened strips.
- * (c) if in any column each number is less than $N/2$. In such case the candidate to whom that column refers is placed last, and his row and column are covered with fastened strips.
- * (d) if now in any column each uncovered number is less than $N/2$. In such case the candidate to whom that column refers is placed second last, and his row and column are covered with fastened strips.

*These processes may be continued as often as possible.

9. When as many candidates as possible have been placed by (8), the uncovered numbers in each column are summed. The column with the lowest sum and the corresponding row are covered with unfastened strips (see section II below), and the uncovered numbers in each column again summed. The column with the lowest sum at this stage and the corresponding row are then covered with unfastened strips, and this process of summing and covering is repeated until only two columns are left uncovered. Of the candidates to whom these columns refer that one who has the majority of preferences over the other shall be placed next in order of those elected. Removing all the unfastened strips, his row and column are now covered by fastened strips.
10. *Again inspect to see if any further candidates can be placed by inspection as in (8). If so, cover the corresponding rows and columns by fastened strips.

11. Proceed again by summing and covering to place another candidate, and so on until a sufficient number have been placed to enable the successful candidates to be determined.

It is to be noted that as each candidate is definitely placed, his row and column are covered with *fastened strips*, but, when a row and column are to be temporarily excluded in the process of placing another candidate, they are covered with *unfastened strips*.

12. If at any stage it is necessary to discriminate between two columns (for instance, those referring to R and S) which have the same sum, then the column S shall be excluded if R has a majority of the preferences as between R and S, but if R has exactly half the preferences as between R and S, the Returning Officer shall decide which column is to be excluded. If it is necessary to discriminate between three or more columns which have the same sum, the Returning Officer shall decide which column is to be excluded.

13. The most complete check will be given by making an independent tabulation in precisely the same way as the first was done. The two are compared, and where any difference is found the voting papers are gone through, the preferences as between each pair of candidates for whom there is a difference being counted. If the result agrees with the result in one of the tables, it may be assumed that this result is correct. If, however, it agrees with neither, a second count for this pair of candidates may be made, and so on until reasonable certainty has been secured.

As a general rule, however, it will probably not be necessary to carry out the complete retabulation, but a count of the preferences as between two candidates should be carried out wherever it appears possible that a slight error in the tabulation could affect the election. The most usual case will be where, when the candidates are reduced to two, each of them has approximately half the preferences over the other.

V.

Actually at the recent University elections there were 1550 voting papers and 6 candidates for the Council, and 688 voting papers and 9 candidates for the Standing Committee. Dealing with the papers occupied some 12 scrutineers about 13 hours, one-third of which was spent in opening the envelopes and checking the names of the voters.

None of those present except the Returning Officer (Mr. Bainbridge) and myself had had any previous experience of the tabulation. No difficulty whatever was found with the ordinary

papers, but a little preliminary practice was shown to be advisable with voting papers which had bracketing or were incomplete, so that the treatment of them may become mechanical.

The above figures enable an estimate of the time necessary for obtaining the result of any election by this system to be made. In round numbers a pair of scrutineers without previous experience can deal with 1000 preferences per hour. The number of preferences on a voting paper is $n(n-1)/2$, where n is the number of candidates. If then N is the number of voting papers, and m the number of scrutineers, the time taken would be approximately $N.n(n-1)/1000m$ hours.

VI.

PROCEDURE WHERE NUMBER OF CANDIDATES IS LARGE.

It will be seen that the amount of work involved increases rapidly with the number of candidates, and when the number of candidates exceeds 10 may well become too great. When there are more than 10 candidates, the procedure described below will enable the result to be obtained with almost mathematical certainty, while at the same time reducing to 10 the number of candidates whose preferences are to be tabulated.

Since the object of the election is simply to elect a certain number of candidates, and not necessarily to place them in order, it will be found that for a candidate to be elected by the proposed short cut who would be rejected by Nanson's system, or vice versa, requires that the position of such a candidate must differ by *at least* six places from the position he would occupy if the Nanson system were adopted in its entirety. The likelihood of this happening is so infinitesimal that it may be disregarded. The procedure recommended is as follows:—

1. Whenever the number of candidates exceeds 10, and the number of vacancies exceeds 5, make use of the method⁴ described in (2) below to elect a certain number of candidates. If the excess of the number of candidates over the number of vacancies—
 - (a) is 5 or more, the number to be so elected is equal to the number of vacancies in excess of 5;
 - (b) is less than 5, the number to be so elected is equal to the number of candidates in excess of 10.
2. Determine the number of first preferences allotted to each candidate. For each paper where any number p of candidates are placed equal with a preference ranking as first, $1/p$ is to be credited to each of the candidates so placed.

4.—Ware's method, modified to allow of bracketing of candidates.

3. Exclude provisionally the candidate with the lowest number of first preferences so determined, and determine by the above rule the number of preferences ranking as first preferences allotted to each remaining candidate. Continue this process of provisional exclusion of the candidate with the lowest number of preferences ranking as first preferences and the determination of the number of preferences ranking as first preferences allotted to each remaining candidate until one only remains. This candidate is declared elected, and is excluded from further scrutiny.
4. Including those provisionally excluded, determine the number of preferences ranking as first preferences allotted to each remaining candidate, and proceed as in (3), excluding provisionally the candidates one by one until one only is left, who shall be declared elected, and excluded from further scrutiny.
5. Repeat the process in (4) as often as is necessary to elect the number of candidates given by (1).
6. At any stage a candidate who has an absolute majority of preferences ranking as first preferences shall be declared elected forthwith, and excluded from further scrutiny.
7. If, after the election of such candidates (if any) as shall be elected under (1), the number of remaining candidates is greater than 10, the number shall be reduced to 10 by the rejection of candidates as follows:—
Determine by the rule in (2) the number of preferences ranking at this stage as first preferences allotted to each of the remaining candidates. Exclude provisionally the candidate with the highest number of preferences ranking as first preferences, and again determine the number of preferences, ranking as first preferences, allotted to each of the remaining candidates. Continue this process of provisional exclusion of the candidate with the highest number of preferences ranking as first preferences, and the determination of the number of preferences ranking as first preferences allotted to each remaining candidate until one only remains. This candidate is declared rejected, and is excluded from further scrutiny.
8. Including those provisionally excluded, determine the number of preferences ranking as first preferences allotted to each remaining candidate and proceed as in (7), excluding provisionally the candidates one by one until one only is left, who shall be declared rejected, and excluded from further scrutiny.
9. Repeat the process in (8) until the number of remaining candidates has been reduced to 10.

10. If at any stage under (3) to (9) it be necessary to discriminate between candidates with an equal number of preferences that rank as first preferences at that stage, the Returning Officer shall have a casting vote.

Note.—In the above, 10 and 5 may be replaced by $2n$ and n respectively, where $2n$ is the number of candidates that can be dealt with conveniently under the Nanson system.

VII.

As the technique of Ware's method is well established, it is unnecessary to give here any instructions for its use, and the method of dealing with bracketed votes is clear from Instructions (1) and (2) earlier in this paper. At each stage each group of papers should be labelled to show to which candidate or candidates first preference was given, and which candidates have preferences ranking at that stage as first. This will render the counting of bracketed preferences an easy matter, and also will facilitate redistribution.

When the number of candidates has been reduced to 10 as described above, further dealing with the voting papers will be facilitated by placing over the voting paper a card which has been cut so that only those portions of the paper referring to the 10 remaining candidates can be seen.

VIII.

In conclusion, it cannot be too strongly emphasised that the Nanson system is *not* a system of allotting *marks*. In it the voting paper is simply and solely a means of showing which one of each pair of candidates the voter prefers, and the method of tabulation adopted shows at a glance how many voters preferred P to Q. The candidates are eliminated one at a time, until two only, P and Q say, remain.

If a voter expressed no preference as between P and Q, either by bracketing them, or by not placing a number against their names, this has been indicated by a dot in column P, row Q, and a dot in column Q, row P. Each dot has been counted as half a preference, but as there is the same number of dots in each of the two spaces which show P's preferences over Q, and Q's preferences over P, the dots exactly balance, and it is immaterial what value is assigned to a dot as far as the contest between P and Q (or any other pair of candidates) is concerned.

When no candidate has a majority over each of the other candidates, it is necessary to adopt some criterion for the exclusion of a candidate. The criterion adopted in the above method is that the candidate with the lowest number of preferences shall be provisionally excluded. In order that each voting paper should have equal weight in this exclusion, it is necessary, where no preference has been expressed as between two candidates, to

credit each of the two candidates with half a preference. Incidentally, the use of the dot reduces the work of tabulation to less than one-half, as with it the detailed tabulation need only be made on one side of the diagonal.

In Ware's method, the criterion is that the candidate with the lowest number of first preferences shall be provisionally excluded, and the rule adopted above for counting bracketed preferences is that necessary for giving equal power to each voter. Owing to this difference in the criterion adopted, Ware's method may lead to the anomalous result that a candidate may obtain a majority of preferences as against each other candidate, and yet not be elected. (Nanson, *loc. cit.*) Thus Ware's method does not fulfil the fundamental condition which a true majority system must fulfil. Under Nanson's method such a candidate would *always* be elected, so that Nanson's method is theoretically sound, and, if the scrutiny is carried out as described in this paper, is also a method which can be readily applied in practice.

END OF VOLUME XXXIX., PART I.

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